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MOLECULAR MEDIUM AND A FREE ELECTRON LASER

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ABSTRACT

The system of a relativistic electron beam and an active molucular (maser) medium in the presence of a ripple magnetic field is studied theoretically. The tunability of the free electron laser could be used to reach different lines of such a molecular medium. We use a Lorentz model to describe the molecular transitions, a hydromagnetic model for the beam, and formulate the resonance condition for the two cases where (1) the coupling occurs at a common interface, e.g., cylindrical, between the beam and the molecular medium, and (2) the beam and the molecules fill the same volume in space.

The coupling between the beam and the molecules offers possibilities of enhanced radiation and improved coherence. In the case where the free electron laser acts as a pump of the molecules stimulated radiation could in principle be obtained at a new wavelength, i.e., the coupled system acts as a frequency transformer.

INTRODUCTION

Theoretical studies have recently been made of the properties of active molecular plasma media, i.e. media where a plasma is doped with molecules which have inverted populations of their energy levels [1-5]. Such media have free energy available due to the inversion of populations. As a consequence, new and interesting wave interactions occur between molecular maser and plasma wave types.

An intense research is going on in radio astrophysics to study spectral line radiation from interstellar gas clouds which radiate in the cm and mm wave domains [6-9] and future research will be extended into the submillimeter wave region. Cosmic masers clearly exhibit a variety of lines in these wavelength regions [6]. Infrared pumping of lines seems to play an interesting role in this connection [9].

It seems, however, that only under particular circumstances the plasma densities are so high that any appreciable coupling between the plasma and the maser radiation occurs.

For significant coupling to occur between an electron plasma and the radiative transitions of the molecules (or ions) it should be most interesting to consider a relativistic electron beam [10] rather than a stationary electron plasma. A free electron laser system (FEL), where a ripple magnetic field in the transverse direction couples the beam waves to transverse radiation [11-22] would, it seems, be particularly interesting in this respect. The tunability of the free electron laser, which is a main advantage of the FEL system, could be used to reach different lines of an active molecular medium. As one possibility, energy could then, in principle, be tapped by a stimulated process from the free energy available in the inverted population of the lines of the molecular medium and transformed into enhanced radiation. Alternatively, the

FEL radiation could act as a pump for the molecules. Stimulated radiation from the molecules could then, in principle, also be obtained with a different wavelength if the de-excitation occurs to other molecular levels. In this way, FEL energy could even be transformed into radiation with improved coherence. Ideas of this kind led the author to start the present study, which was primarily intended to be of fundamental interest. It seems, however, that the idea of coupling the active molecular medium to a free electron laser needs further consideration in view of new possibilities for future practical applications. Similar possibilities may apply also to gyratron radiation.

Basic Equations for Waves in an Active Molecular Medium in the Presence of a Rippled Transverse and a Guiding Longitudinal Magnetic Field

We consider a homogeneous infinitely extended medium of molecules (or ions) with inverted populations of their energy levels, i.e. a maser system. An external rippled transverse magnetic field and a guiding magnetic field is imposed, and can be described as follows [20],

$$\bar{B}_o = B_{oz}\hat{e}_z + \delta B_o\,(\hat{e}_x \cos k_o z + \hat{e}_y \sin k_o z) \qquad (1)$$

where B_{oz} and δB_o are constants which measure the strength of the homogeneous and rippled magnetic fields, respectively.

The maser system we describe by using a damped Lorentz oscillator model which considers only two energy levels, E_1 and E_2, and for which the population inversion is regarded as constant [2].

We now introduce the notations $\bar{\xi}$ and $\bar{u}$ for the mean displacement and velocity of the bound electrons relative to a molecular frame, $N_m = N_1 - N_2$ for the population difference (N_m is negative in active molecular medium) and furthermore assume that $\omega_r = (E_2 - E_1)/\hbar$ is the transition frequency; μ_m is the transition element; $g_m = 2\omega_r|\mu_m|^2 m_-/(3\hbar q_-^2)$ is the oscillator strength; $\omega_m^2 = g_m N_m q_-^2 / (m_-\varepsilon_o)$, with the notations q_- and m_- for the charge (absolute value) and mass of the electrons, ω_m is the molecular plasma frequency; $\omega_{cm} = q_- B_o/m_-$ is the Zeeman split for the molecular system and $\omega_{c\delta} = q_-\delta B_o/m_-$ is the corresponding split caused by the rippled magnetic field; ν_m is the damping of the molecular system. We assume a dependence on space and time of each oscillating variable as given by $\exp(ikz - i\omega t)$

From the equation of motion, we have

$$\frac{\partial \bar{u}}{\partial t} + \nu_m \bar{u} + \omega_r^2 \xi + \frac{q_-}{m_-} \bar{u} \times \bar{B}_o + \frac{q_-}{m_-} \bar{E} = 0 \tag{2}$$

where $\bar{u} = \partial \bar{\xi}/\partial t$. Thus we obtain the equation

$$\frac{\partial^2 \bar{\xi}}{\partial t^2} + \nu_m \frac{\partial \bar{\xi}}{\partial t} + \omega_r^2 \bar{\xi} + \frac{q_-}{m_-} \frac{\partial \bar{\xi}}{\partial t} \times \bar{B}_o + \frac{q_-}{m_-} \bar{E} = 0 \quad . \tag{3}$$

From Maxwell's equations

$$\nabla \times \bar{E} = -\mu_o \frac{\partial \bar{H}}{\partial t} \tag{4}$$

$$\nabla \times \bar{H} = \varepsilon_o \frac{\partial \bar{E}}{\partial t} + \bar{j}_m \tag{5}$$

where the molecular polarization current is

$$\bar{j}_m = -q_- \, g_m \, N_m \, \bar{u} \quad . \tag{6}$$

Combining Eqs. (4-6) we obtain the equation

$$\nabla \times \nabla \times \bar{E} = -\mu_o \varepsilon_o \, \partial^2 \bar{E}/\partial t^2 + \mu_o q_- \, g_m \, N_m \, \partial^2 \bar{\xi}/\partial t^2 \quad . \tag{7}$$

Introducing the notations

$$E_{\pm} = E_x \pm i \, E_y \tag{8}$$

for the right- and left-hand circularly polarized waves and correspondingly for the transverse displacements

$$\xi_{\pm} = \xi_x \pm i \, \xi_y \tag{9}$$

we then have from Eq. (7)

$$(k^2 - \omega^2/c^2)\, E_{\pm} + \mu_o q_- \, g_m \, N_m \, \omega^2 \, \xi_{\pm} = 0 \tag{10}$$

and

$$\varepsilon_o E_z - q_- \, g_m \, N_m \, \xi_z = 0 \tag{11}$$

for the transverse and longitudinal components, respectively. Substituting the electric field components from Eqs. (10) and (11) in Eq. (3) we obtain the following equations for the components of the molecular electron displacements

$$\begin{aligned}[\omega_r^2 - \omega^2 \mp \omega_{cm} - i\nu_m\omega - (\omega^2/c^2) \cdot \omega_m^2/(k^2 - \omega^2/c^2)]\xi_{\pm} \\ \pm \omega_{c\delta}\, \omega \exp(\pm ik_o z)\xi_z = 0\end{aligned} \tag{12}$$

and

$$\begin{aligned}(\omega_r^2 + \omega_m^2 - \omega^2 - i\nu_m\omega)\xi_z \\ - \frac{1}{2}\, \omega_{c\delta}\omega[-\xi_+ \exp(-ik_o z) + \xi_- \exp(ik_o z)] = 0\ .\end{aligned} \tag{13}$$

Assuming the transverse magnetic ripples to be small, i.e. $\omega_{c\delta}$ small, we now expand

$$\bar{\xi} = \exp(ikz - i\omega t) \sum_{n=-\infty}^{\infty} \bar{\xi}_n \exp(ink_o z) \tag{14}$$

and keep only terms up to $n = \pm 1$.

We then obtain the following two coupled equations

$$[\omega_r^2 - \omega^2 \mp \omega_{cm}\omega - i\nu_m\omega - (\omega^2/c^2)\omega_m^2/(k^2 - \omega^2/c^2)]\xi_{\pm,k} \pm \omega_{c\delta}\omega\xi_{z,k\mp k_o} = 0 \tag{15}$$

$$(\omega_r^2 + \omega_m^2 - \omega^2 - i\nu_m\omega)\xi_{z,k\mp k_o} \mp \frac{1}{2}\omega_{c\delta}\omega\ \xi_{\pm,k} = 0 \quad . \tag{16}$$

From Eqs. (15) and (16), we can then write the dispersion relation in the following form

$$(\omega_r^2 + \omega_m^2 - \omega^2 - i\nu_m\omega)[\omega_r^2 - \omega^2 \mp \omega_{cm}\omega - i\nu_m\omega - (\omega^2/c^2)\omega_m^2/(k^2 - \omega^2/c^2)] = -\frac{1}{2}\omega_{c\delta}^2\ \omega^2 \tag{17}$$

Here, the term on the right hand side represents the effect of the rippled magnetic field, which as is clearly seen from Eq. (17) couples the longitudinal and transverse waves in the active molecular medium.

Characteristic Waves of the Free Electron Laser and their Resonant Coupling to the Waves in the Active Molecular Medium

The free electron laser uses a relativistic electron beam to produce coherent electromagnetic radiation. The coupling between the beam electrons and the generated radiation field is caused by a transverse rippled or helical magnetic field. As seen from a frame of reference where the electron beam is at rest the electrons experience the rippled field as a wave, which becomes scattered by the beam to produce the radiation field. The scattering is of the stimulated Raman or Compton backscattering type [21], which has recently been extensively studied in connection with laser-plasma interaction in laser pellet coronas.

The wavelength of the radiation which the electron beam produces when it passes through the rippled magnetic field is roughly given by $\lambda_o/2\gamma^2$ for large values of the relativistic factor γ, where λ_o is the wavelength of the rippled magnetic field. By varying γ, i.e. the beam velocity, or the wavelength of the rippled magnetic field the free electron laser can therefore be tuned to cover in principle a wide domain of wavelengths for example in the millimeter or infrared wave regions.

Let us regard the characteristic waves of the free electron laser, neglecting for simplicity the effect of the rippled magnetic field, but including the influence of a guiding static magnetic field in the direction of propagation, which we take to be the z-axis. The dispersion relation for the longitudinal wave [10,17,20]

$$\omega = kv_{oz} \pm \omega_{pe}/\gamma^{3/2} \tag{18}$$

where thermal effects have been neglected and where ω_{pe} is the plasma frequency of the electron beam.

The electromagnetic waves propagating along the guiding external magnetic field are governed by the following dispersion relation [10,20]

$$\omega^2 = k^2c^2 + \frac{\omega_{pe}^2\,(\omega - kv_{oz})}{\gamma(\omega - kv_{oz} \mp \omega_{ce}/\gamma)} \tag{19}$$

where ω_{ce} is the electron cyclotron frequency and minus (plus) sign refers to the right (left) hand polarization of the electromagnetic wave.

We are here particularly interested in the waves of large ω and k values. The main asymptotic features of the free electron laser wave dispersion diagram are then governed by the relation for the right hand polarized cyclotron wave

$$\omega \simeq kv_{oz} + \omega_{ce}/\gamma \tag{20}$$

and that for the electromagnetic wave, which corresponds to $|\omega - kv_{oz}| >> \omega_{ce}/\gamma$, namely

$$\omega^2 = k^2c^2 + \omega_{pe}^2/\gamma \tag{21}$$

Of the two possibilities of Eqs. (20) and (21) the cyclotron wave of (20) is likely to be heavily damped due to resonance with spiraling electrons. Therefore, an electromagnetic wave satisfying Eq. (21) is the foremost candidate to be considered to produce a coupling to the active molecular medium.

For practical reasons, i.e. to avoid complications due to scattering of beam electrons by the gas molecules, we do not consider here the interpenetration of the beam in the molecular medium, but assume instead that the coupling occurs electromagnetically through a common interface parallel with the beam direction (in an experiment preferably cylindrical).

A condition for resonant coupling between the electromagnetic waves associated with the relativistic electron beam and the active molecular medium, respectively, would then be that the frequency and wave-numbers are the same for the waves in the two media.

Neglecting in Eq. (17) the coupling caused by the ripple field, which should be an approximation valid when the frequency ω is not too close to the resonance $\omega^2 \simeq \omega_r^2 + \omega_m^2$ (notice the combined nature of single particle and collective wave motion corresponding to this resonance), we have from Eq. (17)

$$\omega^2 = k^2c^2 - \omega^2 \frac{\omega_m^2}{\omega_r^2 - \omega^2 \mp \omega_{cm}\omega - i\nu_m\omega} \tag{22}$$

Disregarding furthermore the damping ν_m (caused mainly by coupling to other molecular lines) the resonance condition could be expressed in the simple form

$$\frac{\omega_{pe}^2}{\omega^2\gamma} = \frac{|\omega_m^2|}{\omega_r^2 - \omega^2 - \omega_{cm}\omega} \quad ; \quad \omega_m^2 < 0 \tag{23}$$

where we have chosen the right hand polarized wave also for the active molecular medium wave, and where we notice that for inverted population i.e. maser amplification, we have $\omega_m^2 \sim N_1 - N_2$ negative.

To satisfy relation (21) it seems necessary that (for the case here considered)

$$\omega^2 < \omega_r^2 - \omega_{cm}\omega ,$$

i.e. for weak guiding magnetic fields $\omega^2 \leq \omega_r^2$.

The electron beam velocity should be used for frequency tuning to adjust γ to the proper value for resonance as given by Eq. (21), where the ordering of parameters given by $|\omega_m^2| < \omega_{pe}^2 < \omega_r^2$ is common in practical cases.

The Dispersion and Growth of Resonant Radiation in an Active Molecular Medium Interpenetrated by a Relativistic Electron Beam

Let us now consider the physical situation where an active molecular medium is interpenetrated by a REB, i.e., the beam and the medium are present in the same volume in space. Considering as before high values of ω and k, we then have the following relation for the dispersion of transverse electron-magnetic waves

$$\omega^2 = k^2c^2 + \frac{\omega_{pe}^2(\omega-kv_{oz})}{\gamma(\omega-kv_{oz} \mp \omega_c/\gamma)} - \omega^2 \frac{\omega_m^2}{\omega_r^2 - \omega^2 \mp \omega_c\omega - i\nu_m\omega} \qquad (24)$$

We may regard this relation (24) as the asymptotic e m branch of the corresponding FEL dispersion diagram (ripple field and transverse geometry effects neglected). For simplicity we here also take $|\omega-kv_{oz}| >> \omega_c/\gamma$ and $|\omega_c| < \nu_m$. The dispersion relation then becomes

$$\omega^2 = k^2c^2 + \omega_{pe}^2/\gamma - \omega^2 \frac{\omega_m^2}{\omega_r^2-\omega^2-i\nu_m\omega} \qquad (25)$$

From (25) we now determine the fastest growing wave by varying ω, which we regard as real, and calculating the value of ω for which $\partial/\partial\omega(\mathrm{Im}k) = 0$. We set $k = \mathrm{Re}(k) - i\Gamma$ and notice for convenience of the calculation that we can use the relation $\partial/\partial\omega(\mathrm{Im}k) = \mathrm{Im}(\partial k/\partial\omega) = 0$. We furthermore introduce the quantity Δ for the shift of ω from molecular resonance, ω_r, so that

$$\omega = \omega_r - \Delta$$

We then obtain from (25)

$$\Gamma \simeq -\frac{1}{2}\,\mathrm{Re}(k)\,\frac{\omega_m^2}{\omega_r}\,\nu_m\,\frac{\nu_m^2 + 8\omega_r\Delta}{[\nu_m^2+4\Delta^2]^2 - \omega_m^2[\nu_m^2-4\Delta^2]} \qquad (26)$$

where we have neglected terms of order Δ/ω_r compared to unity.

We also find from (24) for the Re(k)

$$\mathrm{Re}(k) \simeq \frac{\omega_r}{c}\left(1 - \frac{\omega_{pe}^2}{\gamma\omega_r^2} + 2\,\frac{\omega_m^2\,\Delta/\omega_r}{4\Delta^2 + \nu_m^2}\right)^{1/2} \tag{27}$$

For a medium with inverted populations ($\omega_m^2 < 1$; $N_2 > N_1$) we have from (25) and (26)

$$\Gamma \simeq \frac{1}{2}\frac{1}{c}\left(1 - \frac{\omega_{pe}^2}{\gamma\omega_r^2} - 2\,\frac{|\omega_m^2|\,\Delta/\omega_r}{4\Delta^2+\nu_m^2}\right)^{1/2} |\omega_m^2|\,\nu_m\,\frac{\nu_m^2 + 8\omega_r\Delta}{(\nu_m^2+4\Delta^2)^2+|\omega_m^2|\,(\nu_m^2-4\Delta^2)} \tag{28}$$

Relation (28) expresses the growth-rate of the fastest growing transverse wave in a system of a REB and a molecular medium. So far we have not considered the presence of a rippled magnetic field in this section. If we now consider separately the wavelength of free electron laser radiation in the REB and active molecular medium we have for large γ^2 that

$$\lambda \simeq \lambda_o\,[n-1+1/(2\gamma^2)]$$

where we have taken into account the fact that the radiation is propagating in the medium with an index of refraction n, where

$$n \simeq \left(1 - \frac{\omega_{pe}^2}{\gamma\omega_r^2} + 2\,\frac{\omega_m^2\Delta/\omega_r}{4\Delta^2 + \nu_m^2}\right)^{1/2} \tag{29}$$

Thus the transverse wave created by the rippled magnetic field will have

$$\mathrm{Re}(k) = k_o\,[1/(2\gamma^2)+n-1]^{-1} \tag{30}$$

with n given by (29). For this wave to be resonant with the fastest growing wave in the combined medium of the REB and the active molecular gas the expression (30) should be equal to the Re(k) as given by (27). To lowest order we here neglect the influence of the coupling by the ripple magnetic field [compare relation(17)] on Re(k) and find from (27) and (30) the following resonance condition

$$n[n-1+1/(2\gamma^2)] = k_o c/\omega_r \tag{31}$$

Assuming that in (29) $n \simeq (1+\varepsilon)^{1/2} \approx 1+\frac{1}{2}\varepsilon$ where ε is small we find to lowest order in ε from (31)

$$1/(2\gamma^2) - \frac{1}{2}\frac{\omega_{pe}^2}{\gamma\omega_r^2} - \frac{|\omega_m^2|\Delta/\omega_r}{4\Delta^2+\nu_m^2} = k_o c/\omega_r \tag{32}$$

where we have assumed $\omega_m^2<0$, $N_1<N_2$.

Relation (32) indicates how resonant coupling can be achieved in this case by tuning the REB drift velocity.

Under resonant conditions we expect an appreciable coupling to occur between the beam and the molecular medium with a transverse wave total growth-rate of the order of the sum of Γ as expressed by (28) and an additional part corresponding to that of the free electron laser [17].

An interesting interplay between the free electron laser radiation and the excitations of the molecular medium may thus occur and give rise to possible improvements in coherence of the radiation and provide new means for frequency transformations.

Concluding Remarks

From a practical point of view it depends on the parameters of the molecular medium and the beam, e.g., the gas pressure and electron beam density whether or not it is feasible to let the beam interpenetrate the gas without causing too strong undesirable effects like scattering and ionization. Interaction at a cylindrical interface between separate media remains. A possibility would then be to have the gas in a central cylinder tube surrounded by an annular REB with the external magnetic ripple field generated around the outer beam surface.

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